The multi-connected momentum distribution and fermion condensation

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The multi-connected momentum distribution and fermion condensation

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Abstract. The structure of the ground state beyond the instability point of the quasiparticle system with a Fermi-step momentum distribution is studied within the model of a Fermi liquid with a strong repulsive interaction. A ground-state rearrangement occurs as the interaction strength is increased beyond a certain critical value. Numerical investigation of the initial stage of this structural transition shows that there are two temperature regions, corresponding to different scenarios of the rearrangement. While for temperature T larger than some characteristic temperature T_0 the behaviour of the system is the same as that in the case of fermion condensation, for $T \lesssim T_0$ an intermediate structure with a multi-connected quasiparticle momentum distribution arises.

1. Introduction

The question of the applicability of the Landau Fermi-liquid theory [1] for describing properties of strongly correlated Fermi systems has been under discussion for a long time. It is known that this theory is not valid for one-dimensional (1D) systems [2]. For such systems, the concept of a Luttinger liquid [3] with a single-particle Green function containing no quasiparticle pole is usually introduced, instead of the quasiparticle picture. The boundaries of the non-Fermi-liquid view also encompass strongly correlated 2D liquids [4, 5], since HTC materials with quasi-2D structure possess properties at variance with expectation from the Landau theory. However, the recently measured electronic spectra of such materials [6–8] show evidence for the presence of a quasiparticle pole in the single-electron propagator. At the same time, new possibilities were found within the quasiparticle approach in references [9–12], where quasiparticles with momentum distributions differing from the ones assumed in the Landau theory are introduced. This new class of systems with the presence of a fermion condensate, as predicted in references [11, 12], possesses a rich variety of properties [12–15], some of which are characteristics of a non-Fermi-liquid behaviour. As discussed within different models [12–14, 16], a state with the fermion condensate arises as a result of the rearrangement of the ground state of the system. This rearrangement in a system of quasiparticles, whose momentum distribution has the shape of a Fermi sphere with an occupation number slightly smoothed at $T > 0$, takes place when some parameters are varied and the pertinent stability condition is violated. In the present work we consider the model of a homogeneous Fermi liquid, which also displays a rearrangement of the ground state of the quasiparticle system, and investigate the scenario of the initial stage of the rearrangement.

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2. Theoretical and computational aspects

2.1. The doubly connected Fermi sphere and the fermion condensate

Let us start by recalling the Landau relation between the quasiparticle momentum distribution $n_{\mathbf{p}}(T)$ and the quasiparticle spectrum $\varepsilon_{\mathbf{p}}(T)$:

$$n_{\mathbf{p}}(T) = \left\{1 + \exp\frac{\varepsilon_{\mathbf{p}}(T) - \mu(T)}{T}\right\}^{-1} \tag{1}$$

($\mu(T)$ is the chemical potential), which results from a variational equation $\delta F/\delta n_{\mathbf{p}} = 0$ (F is the free energy of the system) with the usual expression for the entropy [17]. On one hand, equation (1) is simply the Fermi–Dirac quasiparticle distribution over energies. On the other hand, this relationship is an equation for the quasiparticle distribution over the phase space. In fact, the quasiparticle energy is the variational derivative of the ground-state energy functional $E_0[n]$ with respect to the quasiparticle distribution, $\varepsilon_{\mathbf{p}}(T) = \delta E_0/\delta n_{\mathbf{p}}(T)$, and, therefore, it is itself a functional of $n_{\mathbf{p}}(T)$.

It is postulated in the Landau theory that in a homogeneous isotropic Fermi liquid, like in a Fermi gas, the quasiparticle momentum distribution at $T = 0$ has the shape of a fully occupied Fermi sphere $n_F^{(0)}(p) = \theta(p_F - p)$ (the maximum momentum p_F is related to the density ρ of the system by the well known relation $\rho = p_F^3/(3\pi^2)$). The low-temperature behaviour of the quasiparticle spectrum corresponding to such a momentum distribution has the form [17]

$$\varepsilon_p(T) - \mu(T) = \xi(p) + \mathrm{O}(T^2). \tag{2}$$

The function $\xi(p)$ increases monotonically in the vicinity of the Fermi momentum and changes its sign at $p = p_F$. Its slope at this point, which is the group velocity of the quasiparticles on the Fermi surface $v_F = \mathrm{d}\xi(p)/\mathrm{d}p|_{p=p_F}$, is determined by one of the phenomenological parameters of the Fermi-liquid theory, the effective mass $M^* = p_F/v_F$.

In a strongly correlated Fermi system, a quasiparticle momentum distribution minimizing the energy functional $E_0[n(p)]$ at $T = 0$ may be located away from the corner point $n_F^{(0)}(p)$ of the functional space $[n]$, and the low-temperature behaviour of the corresponding quasiparticle spectrum may differ from equation (2). For instance, it was found in reference [18] that quasiparticle energies which are equal to the chemical potential in a finite region of the momentum space can exist. In references [9,10], some model functionals $E_0[n(p)]$ were introduced which, for certain values of the parameters, reach their absolute minima for a momentum distribution characterized by a doubly connected Fermi sphere:

$$n_F^{(1)}(p) = \theta(p_1 - p) - \theta(p_2 - p) + \theta(p_3 - p). \tag{3}$$

A quite different quasiparticle ground state corresponds to systems with the fermion condensate [11–14]. Let us elucidate the main idea of the concept of a fermion condensate. A homogeneous and isotropic quasiparticle system with the fermion condensate is described by a singular solution of equation (1) which corresponds to a quasiparticle spectrum linear in the temperature T within a finite region of momenta [11, 12]:

$$\varepsilon_p(T) - \mu(T) = T\nu_0(p) + \mathrm{O}(T^2) \qquad p_i < p < p_f. \tag{4}$$

Contrary to the case for the Fermi-liquid formula (2), there is no T-independent term in equation (4). This means that at $T = 0$ the quasiparticle spectrum has a plateau $\varepsilon_p \equiv \mu$ in the region $p_i < p < p_f$. At $T > 0$ the slope of the plateau is linear in T, and its position with respect to the chemical potential $\mu(T)$ is determined by the function $\nu_0(p)$ which is connected with the momentum distribution of quasiparticles in the condensate. Indeed, the singular solution of equation (1), which can easily be obtained upon substitution of the formula (4) into equation (1), has the form $n_p(T) = n_0(p) + \mathrm{O}(T)$, where

$$n_0(p) = \{1 + \exp(\nu_0(p))\}^{-1} \qquad p_i < p < p_f \tag{5}$$

is the momentum distribution of the condensate quasiparticles at $T = 0$. Outside the condensate region, $n_0(p) = 1$ at $p < p_i$, and $n_0(p) = 0$ at $p > p_f$ [11, 12]. One can find the explicit form of $n_p(T)$ and $\varepsilon_p(T)$ provided that one knows the functional dependence $\varepsilon_p(T)[n_p(T)]$. A set of model functionals was suggested in references [12–14, 16], each of them possessing a minimum at the singular solution, within well defined regions of functional parameters. In the present work we indicate the possibility of a scenario in which the transition to the fermion condensate occurs through an intermediate structure, corresponding to a multi-connected quasiparticle momentum distribution.

2.2. The effective functional

The initial stage of the rearrangement, which is studied in this paper, is characterized by variations of the quasiparticle momentum distribution within a relatively thin layer around $p \sim p_F$ (see the results below). Under these conditions one can use the concept of an effective functional, which is widely used in many-body theory. We consider the simple effective functional for the quasiparticle spectrum:

$$\varepsilon_p[n_p(T)] = \frac{p^2}{2M} + \int F(\boldsymbol{p} - \boldsymbol{p}')n_{p'}(T)\,\mathrm{d}\tau' \tag{6}$$

with the effective repulsive interaction

$$F(\boldsymbol{p} - \boldsymbol{p}') = \frac{F_0}{(\boldsymbol{p} - \boldsymbol{p}')^2 + \beta^2}. \tag{7}$$

The symbol $\mathrm{d}\tau$ in equation (6) indicates integration over $\mathrm{d}^3p/(2\pi)^3$ and summation over spin indices. Such a form of effective interaction is often used in calculations for both electronic and nuclear systems. In the discussion below, we use the dimensionless interaction parameter $\gamma = MF_0/(4\pi^2 p_F)$.

The functional dependence given by equations (6) and (7) together with equation (1) and the normalization condition

$$\int n_p(T)\,\mathrm{d}\tau = \rho \tag{8}$$

are the closed set of equations to be solved for the quasiparticle distribution $n_p(T)$ and the spectrum $\varepsilon_p(T)$.

2.3. Numerical aspects

Equation (6), together with expression (1), represents the non-linear integral equation for the function $\varepsilon_p(T)$. This equation was solved numerically by an iterative procedure with weighting factors. The five-point Newton–Cotes quadrature formula with a five-point filter on the output was used for numerically folding the distribution $n_p(T)$ with the effective interaction $F(p, p')$. The momentum grid had a step $h_p = 5 \times 10^{-5} p_F$. The accuracy of the numerical solution was determined by substituting it into the initial equation. The permissible error—that is, the maximum discrepancy between the left- and the right-hand sides of equation (6)—was fixed at $10^{-8}\varepsilon_F$. The number of iterations necessary for reaching this accuracy is about 3×10^4 provided that the iteration weight $w = 0.001$ is taken, which is optimal for stability of the iterative procedure. It is worth noting that the results are independent of which point in the functional space is taken as the initial one for the iterative procedure. For example, the same solution for $\gamma = 0.50$ at $T = 10^{-7}$ was obtained by starting from (i) the solution for $\gamma = 0.50$ at $T = 10^{-5}$ and (ii) the solution for $\gamma = 0.48$ at $T = 10^{-7}$ (here and below the temperature T is taken in units of $\varepsilon_F^0 = p_F^2/2M$).

3. Results

3.1. Instability of the Fermi-step momentum distribution

We begin by estimating the value $\gamma_c^{(0)}$ of the interaction strength at which the necessary condition for stability of the system with the quasiparticle distribution $n_F^{(0)}(p)$ at $T = 0$ is violated. This condition is fulfilled [11, 12] provided that the variation of the ground-state energy E_0 under any admissible variations of the distribution $n(p)$

$$\delta E_0 = \int [\varepsilon(p) - \mu]\delta n(p) \frac{\mathrm{d}^3 p}{(2\pi)^3} \tag{9}$$

is positive. Admissible variations of $n_F^{(0)}(p)$ are of the same sign as the difference $p - p_F$, as dictated by the Pauli principle. Hence, upon substitution of the energy $\varepsilon(p_F)$ for the chemical potential μ in equation (9), one can reformulate the necessary stability condition as a requirement for the value

$$s(p) = 2M\frac{\varepsilon(p) - \varepsilon(p_F)}{p^2 - p_F^2} \tag{10}$$

to be positive for each value of the momentum p [11, 12]. Therefore, if the function $s(p)$ has the first zero close to p_F, the violation of the stability condition means that a bend appears in the curve $\varepsilon(p)$ in the vicinity of the Fermi momentum. To calculate the derivative $\mathrm{d}\varepsilon/\mathrm{d}p$, it is convenient to use the relation of the Fermi-liquid theory [17]:

$$\frac{\partial\varepsilon}{\partial\boldsymbol{p}} = \frac{\boldsymbol{p}}{M} + \int F(\boldsymbol{p} - \boldsymbol{p}')\frac{\partial n}{\partial\boldsymbol{p}'}\,\mathrm{d}\tau'. \tag{11}$$

Substituting the Fermi-step momentum distribution $n_F^{(0)}(p)$ into equation (11), we obtain

$$\zeta^{(0)}(p) = \frac{M}{p_F}\frac{\mathrm{d}\varepsilon}{\mathrm{d}p} = \frac{p}{p_F} + \frac{\gamma p_F}{p} - \frac{\gamma(p^2 + p_F^2 + \beta^2)}{4p^2}\ln\frac{(p + p_F)^2 + \beta^2}{(p - p_F)^2 + \beta^2}. \tag{12}$$

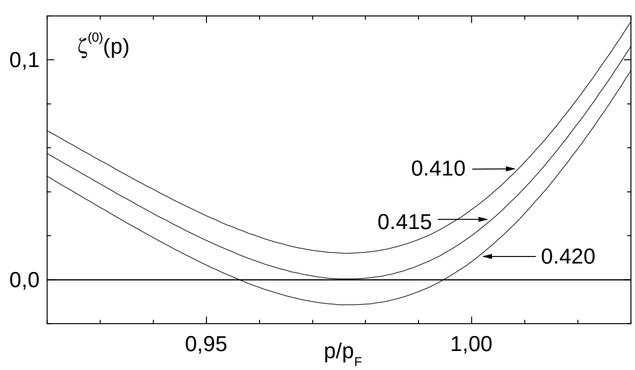

Figure 1. The function $\zeta^{(0)}(p)$ calculated for $\gamma = 0.410, 0.415, 0.420$.

The curves $\zeta^{(0)}(p)$ calculated e.g. for the value $\beta = 0.07p_F$ and different values of γ are shown in figure 1. One can see that the contact with zero takes place at the point $p = p_c \simeq 0.97p_F$ for $\gamma = \gamma_c^{(0)} \simeq 0.415$. Note that the proximity of p_c to p_F justifies the substitution of the function $\zeta^{(0)}(p)$ for the function $s(p)$. Therefore, the ground state with the quasiparticle momentum distribution $n_F^{(0)}(p)$ becomes unstable at $\gamma > \gamma_c^{(0)}$ and its rearrangement takes place. The critical value $\gamma_c^{(0)}$ varies with varying interaction radius β. In figure 2, where the phase diagram in the variables (γ, β) is shown, the curve $\gamma_c^{(0)}(\beta)$ separates the phase of the normal Fermi liquid from the state with the rearranged quasiparticle momentum distribution.

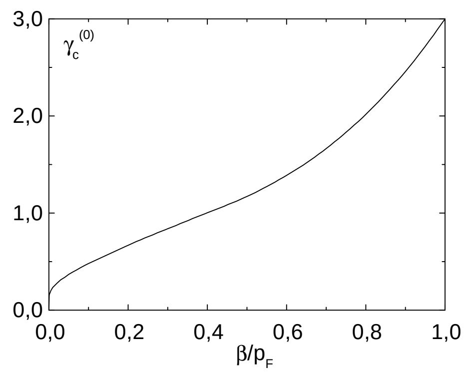

Figure 2. The critical value $\gamma_c^{(0)}$ as a function of the parameter β.

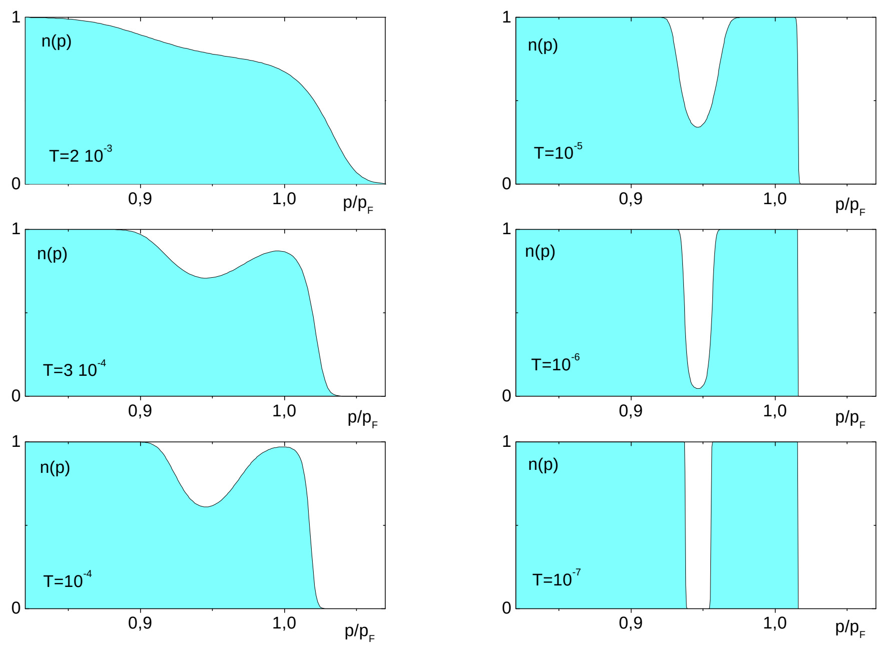

Figure 3. The quasiparticle momentum distributions $n_p(T)$ calculated for $\gamma = 0.45$ at different temperatures.

To get an idea of the physical range of the parameter γ in the case of the Coulomb interaction, we take $F_0 = 4\pi e^2$ and obtain $\gamma = \alpha r_s/\pi$, where $\alpha = (4/9\pi)^{1/3}$. Then we find that for the value $\beta \simeq p_F$ the critical value of the interaction strength $\gamma_c^{(0)} \sim 3$ corresponds to $r_s \sim 18$.

3.2. The multi-connected momentum distribution

We move on to numerical investigation of the rearrangement of the quasiparticle momentum distribution. We study the scenario of this rearrangement with increasing interaction strength γ at fixed value of the interaction radius β. In the present work, we fix the value $\beta = 0.07 p_F$, corresponding to a long-range term of the nuclear effective NN interaction in dense nuclear or neutron matter.

To see how the ground state is arranged just beyond the transition point, let us look at figure 3, where the results of the calculations of $n_p(T)$ for $\gamma = 0.45$ at different values of T are shown. At $T \sim 2 \times 10^{-3}$, the quasiparticle momentum distribution $n_p(T)$ has the shape characteristic of a system with a fermion condensate (we shall discuss this in detail below). At $T \lesssim 2 \times 10^{-3}$ a downturn appears in the distribution, which deepens as temperature decreases, and finally, at $T = 10^{-7}$, one can hardly distinguish $n_p(T)$ from that for the doubly connected Fermi sphere defined in equation (3). The quasiparticle spectrum $\varepsilon_p(T)$ corresponding to such a distribution calculated at $T = 10^{-7}$ is shown in figure 4. The spectrum of a fermion condensate has a shape with a plateau at a value exactly equal to the chemical potential μ at $T = 0$ and beginning to slope slightly at $T > 0$ [11, 12]. Here we have a quite different behaviour, and the quasiparticle spectrum of the doubly connected Fermi-like distribution equals μ at three points, which are at the border p_1 of the inner sphere and the borders p_2 and p_3 of the spherical layer. The deviation of $\varepsilon_p(T)$ from μ reaches the value $\sim 2 \times 10^{-4}\varepsilon_F$ at the point of the minimum of the spectrum and the value $\sim 2 \times 10^{-6}\varepsilon_F$ at the point of its maximum. Despite the latter value being small, it is still higher than the estimated numerical error by two orders of magnitude.

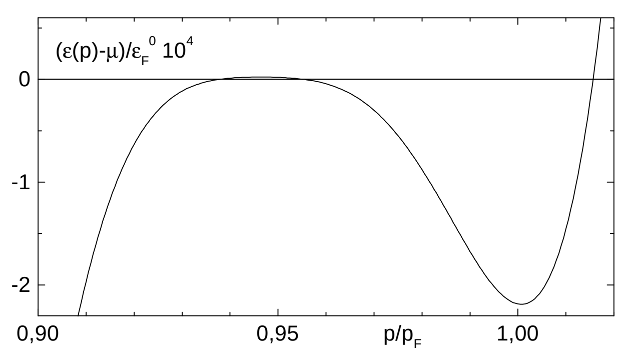

Figure 4. The quasiparticle spectrum $(\varepsilon_p(T) - \mu)/\varepsilon_F^0$ calculated for $\gamma = 0.45$ at $T = 10^{-7}$.

The behaviour of the doubly connected Fermi sphere with increasing interaction strength γ is displayed in figure 5. The spherical layer appearing beyond the transition point has, from the start, a finite thickness, while the gap between the layer and the inner sphere develops starting from a vanishingly small width. The outer layer gets thicker and moves away from the inner sphere as the parameter γ increases. What happens then with such a distribution? To understand that, let us investigate the stability of such a Fermi sphere divided into two layers.

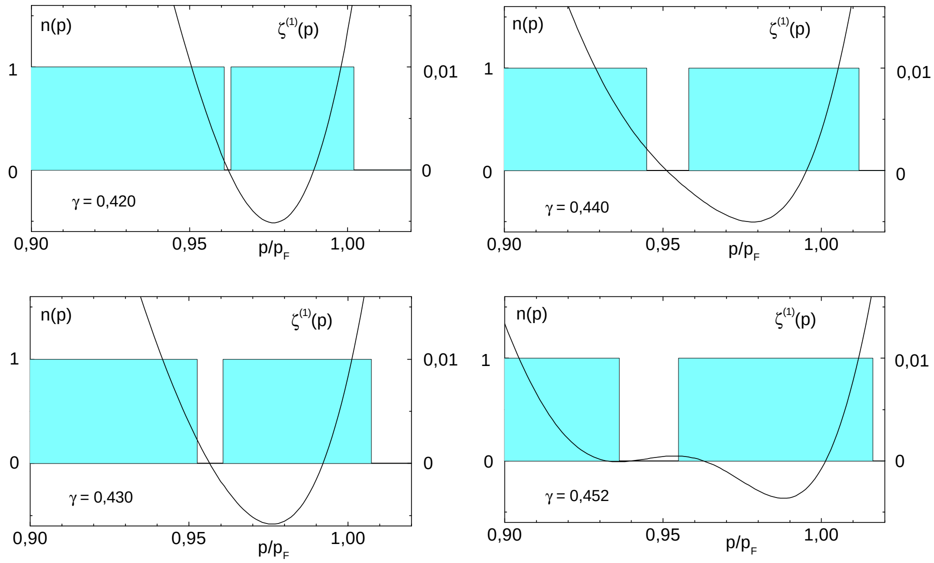

Figure 5. The quasiparticle momentum distributions $n(p)$ and the function $\zeta^{(1)}(p)$ calculated for different values of the parameter γ.

We calculate the function $\zeta^{(1)}(p)$ for the momentum distribution in the form of equation (3) and see when and where the critical change of the sign of that function occurs. Using again relation (11) we find that

$$\zeta^{(1)}(p) = \frac{M}{p_F}\frac{\mathrm{d}\varepsilon}{\mathrm{d}p} = \frac{p}{p_F} + \sum_{i=1}^{3}(-1)^{i-1}\left\{\frac{\gamma p_i}{p} - \frac{\gamma(p^2+p_i^2+\beta^2)}{4p^2}\ln\frac{(p+p_i)^2+\beta^2}{(p-p_i)^2+\beta^2}\right\}. \quad (13)$$

The function $\zeta^{(1)}(p)$ calculated for different values of γ is displayed in figure 5. The points of the maximum absolute values of the derivative $\mathrm{d}n/\mathrm{d}p$ at $T = 10^{-7}$ were taken as the boundary momenta p_1, p_2, p_3 for that calculation. One can easily see that inside the region $\gamma_c^{(0)} < \gamma < \gamma_c^{(1)} \simeq 0.452$, the two points where $\zeta^{(1)}(p)$ changes sign are located in such a way that the corresponding local minimum and maximum of $\varepsilon(p)$ lie in the domains where $n(p)$ equals 0 and 1 respectively. This means that the sign of the difference $\varepsilon(p) - \mu$ coincides with that of the possible variations $\delta n(p)$ allowed by the Pauli principle. Hence the distribution considered satisfies the stability condition.

However, one can see in figure 5 that at $\gamma > \gamma_c^{(1)} \simeq 0.452$, the situation changes. With the displayed behaviour of the function $\zeta^{(1)}(p)$, there are regions where $\varepsilon(p) - \mu > 0$, but with $n(p) = 1$. This means that the necessary stability condition is violated, since allowed variations $\delta n(p)$ exist which lower the ground-state energy. This results in the second rearrangement of the ground state of the system emerging at $\gamma = \gamma_c^{(1)}$. We show in figure 6 how the quasiparticle distribution is arranged beyond the second transition point $\gamma_c^{(1)}$. For definiteness, the results of the calculations for $\gamma = 0.46$ at different values of T are displayed. The calculations show that the new layer appears for the above-mentioned value of the coupling constant and that the quasiparticle distribution $n_p(T)$ at $T = 10^{-7}$ is very close to that for the triply connected Fermi sphere:

$$n_F^{(2)}(p) = \theta(p_1 - p) - \theta(p_2 - p) + \theta(p_3 - p) - \theta(p_4 - p) + \theta(p_5 - p). \quad (14)$$

The calculation shows that the appearance of new layers with increasing values of the parameter

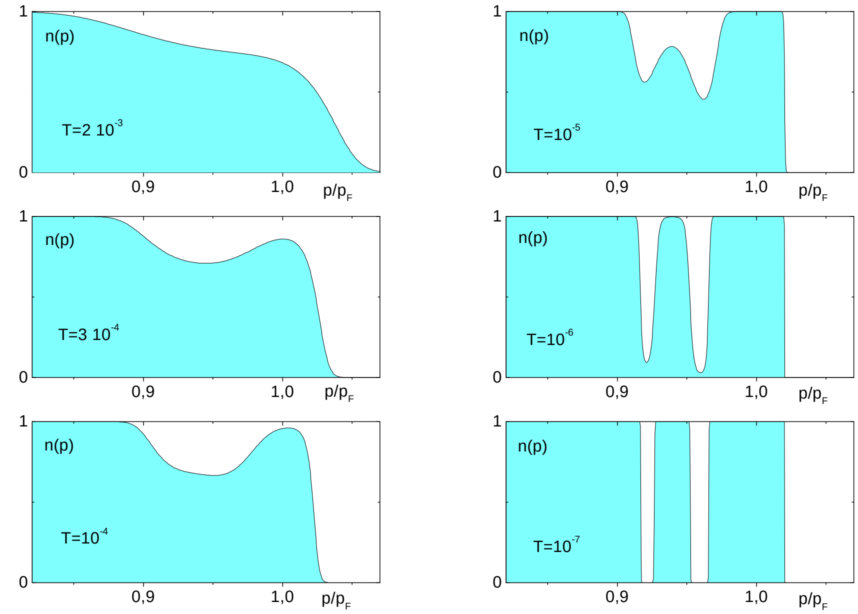

Figure 6. As figure 2, but for $\gamma = 0.46$.

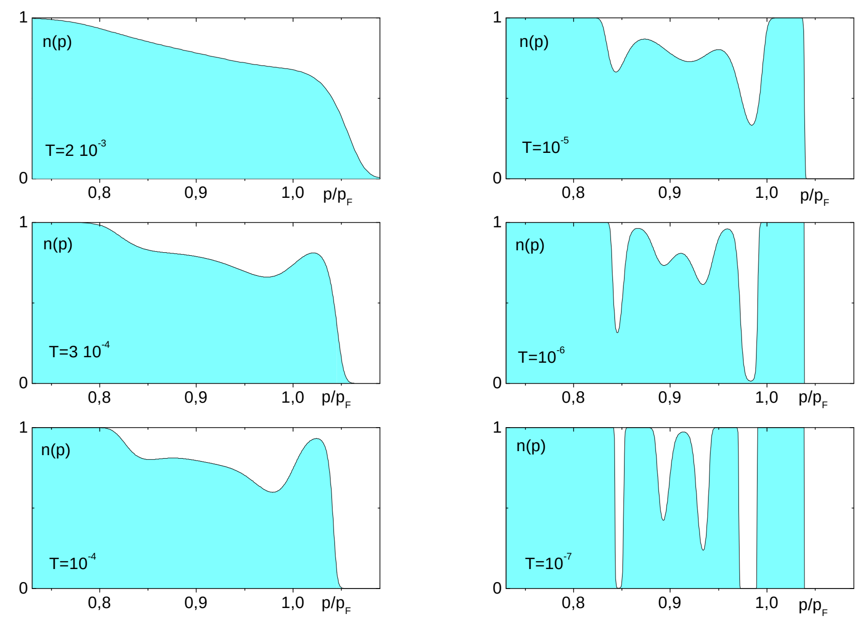

Figure 7. As figure 2, but for $\gamma = 0.50$.

γ does not stop at the level of the triply connected Fermi sphere. Figure 7, where the quasiparticle distribution $n_p(T)$ calculated for $\gamma = 0.50$ is displayed, shows further divisions into layers of the momentum space. The distribution $n_p(T)$ for $\gamma = 0.50$ approaches that of a

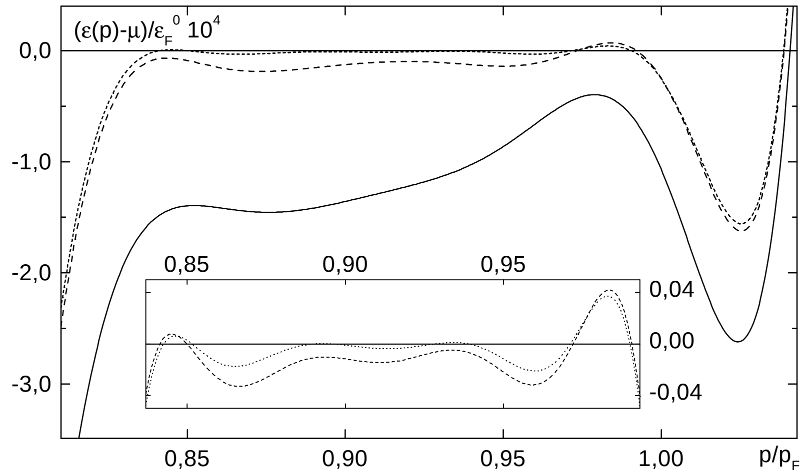

Figure 8. The quasiparticle spectra $(\varepsilon_p(T) - \mu)/\varepsilon_F^0$ calculated for $\gamma = 0.50$ at $T = 10^{-4}$ (solid curve), $T = 10^{-5}$ (long dashes), $T = 10^{-6}$ (short dashes), and $T = 10^{-7}$ (dots).

multi-connected Fermi sphere with lowering T, which is arranged as follows: the small inner occupied sphere with the radius $\sim 0.85 p_F$ is surrounded by four spherical occupied layers with thickness $\sim(0.03–0.04)\, p_F$ divided by spherical empty layers with thickness $\sim(0.01–0.02)\, p_F$. The quasiparticle spectra $\varepsilon_p(T)$ for $\gamma = 0.50$ are shown in figure 8. At $T = 10^{-7}$, the spectrum crosses the line $\varepsilon = \mu$ nine times, at the boundary of the inner sphere and at the boundaries of the spherical layers.

Unfortunately, the computation time sharply increases with increasing phase volume Ω_0 of the layered distribution. This is why the calculations of the present work are carried out up to the value of the interaction strength $\gamma \leqslant 0.5$, corresponding to the initial stage of the rearrangement. The investigation of what happens in the system at larger values of γ is now in progress.

4. Two scenarios of the rearrangement

Thus, beyond the first transition point $\gamma_c^{(0)}$, the scenario of the rearrangement of the quasiparticle ground state at low $T \lesssim 10^{-3}$ is the succession of transitions with increasing γ. Each one of them results in the appearance of a new spherical layer in momentum space. Let us compare this scenario of the rearrangement and some distinctive features of the ground state under consideration with those of the fermion condensation.

Let us recall the main features of the fermion condensation phenomenon. One of them is the plateau in the quasiparticle spectrum $\varepsilon_p(T)$, with a value exactly equal to the chemical potential μ at $T = 0$, in accordance with equation (4), and which develops a finite slope with increasing temperature. Unlike the spectra for systems with the fermion condensate, the spectrum $\varepsilon_p(T = 0)$ for the multi-connected Fermi-like distribution equals the chemical potential at a finite number of points, which are the boundaries of the spherical layers. The low-temperature expansion of such a spectrum has the form of equation (2), typical of the Landau theory, with the non-monotonic function $\xi(p)$ changing its sign several times, unlike the monotonic function for the usual Fermi liquid. At $T = 0$, the ground state with the multi-connected Fermi-like quasiparticle distribution is not macroscopically degenerate, unlike the ground state of a fermion condensate. At the same time, there exist singularities of the density of states connected with the maxima and the minima of the function $\varepsilon_p(T = 0)$. These singularities gradually disappear with increasing T up to $T_0 \simeq 2 \times 10^{-3}$, where the last twist of the spectrum corresponding to the outer layer is smoothed. At $T \gtrsim T_0$ the difference $\varepsilon_p(T) - \mu(T)$ becomes linear in temperature, like that for systems with a fermion condensate.

The other feature of systems with fermion condensates is the shape of the distribution $n(p)$ given by equation (5). Within the region occupied by the fermion condensate, $0 < n(p) < 1$, this corresponds to non-zero entropy of the fermion condensate at $T = 0$. The contradiction with the Nernst theorem disappears, providing that correlations (e.g. superfluidity) are taken into account, which immediately rearrange the ground state due to its degeneracy and re-establish a zero entropy at $T = 0$. The entropy of the state with the multi-connected Fermi-like quasiparticle momentum distribution equals zero at zero temperature because $n(p)$ takes only the values 0 and 1. The multi-layered distribution changes quickly with increasing T: the sharp boundaries of the layers are smoothed and the layers combine together at $T \simeq T_0$ into a monotonically decreasing curve. At $T \gtrsim T_0$ the smoothed $n(p)$ becomes linear in T, like the quasiparticle momentum distribution of a system with the fermion condensate. Being equal to zero at $T = 0$, the entropy of the system with the multi-connected Fermi-like distribution increases sharply with temperature due to the quick smoothing of the function $n_p(T)$. The calculations show that at $T = T_0$ the entropy reaches the value $S_0 \sim \Omega_0/\Omega$, the ratio of the phase volume of the multi-layered region Ω_0 to that of the whole system Ω. Just this value of the entropy would be characteristic at $T \sim 0$ for a system with the fermion condensate occupying the phase volume Ω_0. At $T \gtrsim T_0$ the entropy becomes linear in temperature, like that of a system with a fermion condensate [12, 16].

All of these features of the momentum distribution, entropy, quasiparticle spectrum, and density of states for a system with a multi-connected Fermi-like quasiparticle distribution, as well as the problem of validity of such a quasiparticle pattern, will be studied in detail in a separate paper.

The scenario of the fermion condensation is characterized by the single critical value γ_c of the coupling constant, at which the fermion condensate arises in the system. The phase volume of the fermion condensate increases as γ increases further; however, the shape of the momentum distribution does not modify and no further qualitative changes occur [11, 12]. The scenarios of rearrangement found for the model under consideration are different for different temperatures. At $T = 0$ the scenario is characterized by a sequence of critical values $\gamma_c^{(i)}$, each one corresponding to the appearance of a new ground state with a larger degree of connectivity. The number of critical constants decreases as T increases, so only one of them, $\gamma_c^{(0)}$, survives at $T \gtrsim T_0$. This means that for the model considered, the scenario of formation of the multi-connected Fermi sphere at $T = 0$ gradually transforms into that of the fermion condensation with increasing temperature.

5. Conclusions

In summary, the structure of the ground state of the homogeneous Fermi liquid with strong interparticle repulsion is studied within the framework of the effective-functional approach. The numerical investigation of the simple functional with strong repulsive effective interaction showed that at a fixed value of the interaction radius there exists a critical value of the interaction strength $\gamma_c^{(0)}$, beyond which the ground state with the Fermi-step quasiparticle momentum distribution becomes unstable and the rearrangement of the ground state takes place. The scenario of the initial stage of that rearrangement with increasing γ was found to be different for different regions of temperature. At $T = 0$, there exist a set of critical constants $\gamma_c^{(i)}$ corresponding to the succession of transitions, each resulting in the emerging of a new spherical layer of the quasiparticle momentum distribution $n(p)$). The quasiparticle spectrum $\varepsilon(p)$ corresponding to such a layered distribution does not have a plateau, unlike the fermion condensate, and equals the chemical potential at a finite number of points, which are the boundaries of the layers. The ground state with the multi-connected Fermi-like distribution possesses no macroscopic degeneracy and the entropy of this state is zero at zero temperature. With increasing temperature the layers are quickly smoothed, so at $T \sim T_0 \simeq 2 \times 10^{-3}$ there is no longer any trace of the critical constants $\gamma_c^{(i)}$ with the exception of $\gamma_c^{(0)}$. At $T \gtrsim T_0$ the scenario of the rearrangement is that of the fermion condensation.

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